Atmospheric CO₂ Residence Time and the Carbon Cycle.

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Abstract - An atmospheric CO₂ residence time is determined from a carbon cycle which assumes that anthropogenic emissions only marginally disturb the preindustrial equilibrium dynamics of source/atmosphere/sink fluxes. This study explores the plausibility of this concept, which results in much shorter atmospheric residence times, 4-5 years, than the magnitude larger outcomes of the usual global carbon cycle models which are adjusted to fit the assumption that anthropogenic emissions are primarily the cause of the observed rise in atmospheric CO₂. The continuum concept is consistent with the record of the seasonal photosynthesis swing of atmospheric CO₂ which supports a residence time of about 5 years, as also does the bomb C14 decay history. The short residence time suggests that anthropogenic emissions contribute only a fraction of the observed atmospheric rise, and that other sources need be sought.

1. An Analytic Quandary and an Unorthodox Question

The conventional models described in the literature for the increasing global atmospheric CO₂ have generally assumed that anthropogenic activities have been the source of these increases, and that fossil-fuel use is the principal contributor. This is certainly an understandable assumption, first because such an effect was a historical expectation of the early atmospheric scientists, secondly because the roughly exponential increase in measured atmospheric CO₂ seems to follow a similar increase in fossil-fuel use, and thirdly because the gross amount of fossil-fuel emissions is larger than the atmospheric carbon increase. Based on such circumstantial correlation, the models of the global carbon cycle are customarily fitted to both the atmospheric measurements and estimates of fossil-fuel carbon emitted since the middle 1800s. Past studies of the source-atmosphere-sink dynamics and of the relevant complex ocean behavior have been well reviewed elsewhere.1,2
As the atmospheric CO$_2$ rise is the cause of our concern with climate warming, the CO$_2$ residence time in the atmosphere becomes an important parameter. If it is very long (e.g., a century), then the atmospheric CO$_2$ will continue to accumulate even if major steps are taken to reduce anthropogenic emissions. If it is short (e.g., a few years), then the CO$_2$ growth might possibly be mitigated by reduced emissions. The residence time is derived from the assumption that the rate of diffusion of CO$_2$ from the atmosphere to the sinks is proportional to the mass present, resulting in an exponential decay with a characteristic “residence time”. “Residence time” is defined as the time required for an added mass to be reduced to 37% (1/e) of its initial value and is given by the reservoir content divided by its steady-state flow rate. The real global situation clearly involves a variety of sources and sinks, so many source functions and residence times may be needed to fully describe actual behavior. The present analytical task is to develop a reasonable concept of the principal factors determining atmospheric CO$_2$ residence time in the troposphere.

A comparison of the preindustrial estimates of the carbon cycle with the conventional model studies shows conflicting results on CO$_2$ residence time. The year 1860 preindustrial atmospheric mass is estimated to be about 600 GtC with an annual flow (air-sink flux) through the atmosphere of about 140 GtC/yr, giving a residence time of 4.3 years. This value may be considered typical of the natural steady state, as the fossil-fuel contribution was negligible before then. The 1990 measured atmospheric CO$_2$ is about 750 GtC. The 1990 incremental flow rate from fossil-fuel combustion alone is estimated to be 6.27 GtC/yr. Thus the apparent residence time should now be 750/146=5.1 years. However, if the incremental exponential increase from 1860 to 1990 of atmospheric CO$_2$ (150 GtC) is totally attributed to the accumulated fossil-fuel combustion (238 GtC), then the effective atmospheric residence time for the fossil-fuel contribution would need to be about 61 years (see the Appendix). This anthropogenic residence time component needs to be this long to allow accumulation of the relatively small annual fossil-fuel increment. The effective residence time is the net outcome of the combined action of the many oceanic and terrestrial sinks with various rate characteristics, so it increases slowly with time. For this discussion, it is the magnitude that is significant.

The conventional carbon-cycle models imply that this long residence time results from saturation of the fast-acting sinks at the preindustrial levels, and after 1860 all additional emissions faced a 61 year residence time. This concept of the carbon cycle is explained by Sarmiento and Siegenthaler as based on the assumption that the large (70 - 90 GtC/yr) preindustrial air/sea exchange with the atmosphere is “simply in situ exchange of oceanic and atmospheric CO$_2$ molecules” and is not
Applicable to anthropogenic emissions. This assumed separation leads to the current calculated residence time of about 5 years as the combination of the old short and new long residence times. Thus the usual models suggest that of the 238 GtC emitted, 88 GtC is absorbed by sinks and 150 GtC remains in the atmosphere. If the equivalent residence time is much less than 61 years, more would have gone into the sinks and less into the atmosphere. In contrast, the annual photosynthesis cycle suggests that the large annual flows into both the terrestrial and oceanic sinks are real, involve diffusion into the reacting surfaces, and are not merely "in situ" exchanges. As described later, the observed annual photosynthesis swing in atmospheric CO2 suggests flux magnitudes such as those presented in the various global models. Thus, assuming that saturation of the sinks is absent, then with a preindustrial residence time of 4.3 years, only about 35 GtC would have gone into the atmosphere, and 203 GtC would have gone into the sinks. Thus fossil-fuels would have contributed only 23% of the increase in atmospheric CO2. We are thus left with a quandary, best expressed as follows by Wuebbles and Edmonds:4 "If the fluxes of carbon into and out of the atmosphere generated by natural sources are so large relative to those generated by human activities, it is natural to question whether the observed buildup of CO2 in the atmosphere is simply the outgrowth of natural processes and merely coincident with the emission by human activities. After all, total annual emissions from fossil-fuel use and land-use changes amount to only 3% of the annual natural flux, well within the bounds of uncertainty of the natural flux rates. Nevertheless, circumstantial evidence provides a strong case that human activities are principally responsible for the observed increase in atmospheric CO2."

The preceding observation raises the unorthodox question — can it be possible that a continuing short residence time may be more descriptive of reality than a long residence time, and that fossil-fuel sources are a significant but not dominant source of atmospheric CO2? Is it really possible that circumstantial evidence has misled the model-makers?

2. A Short Residence Time Oceanic Concept

The difficulty of fully modeling the real mix of sources and sinks, particularly the complex, turbulent and somewhat chaotic ocean, is illustrated by the variety of individual author-tailored models that have been proposed (Refs. 1-8). The oceanic sink is usually modeled as a sequence of interacting layers (boxes) in series, whereas the many terrestrial sinks are assumed to act in parallel and somewhat independently.5 The oceanic "boxes" are sometimes arranged as a network. The gross ocean behavior has been simulated by each author from a judgementally crafted assembly of "boxes", each inserted to mimic a set of empirical observations or
to include another variable. This paper will discuss only those features relative to
the atmospheric reservoir and its CO₂ residence time. This is much like describing
the human body by a simple network of "boxes".

The concept in this paper uses the preindustrial atmospheric CO₂ fluxes
suggested by Sundquist of 70 GtC/yr oceanic exchange and 70 GtC/yr for terrestrial
exchange. The historically known fossil-fuel input (1860-1990) is then added to the
preindustrial atmospheric flow, and the relatively small carbon increment is
assumed to redistribute in the ocean sinks and the atmosphere without any
saturation constraints. In order to show the most effect on the oceanic sink, the
terrestrial sinks are assumed not to absorb any of the fossil-fuel contribution.
Obviously, a more complete analysis would include this effect. The dynamics of this
coupled air/sea system is mathematically described in the Appendix. For the
purposes of this illustrative study, the preindustrial residence time of 4.3 years was
assumed for all model calculations in the Appendix.

The important result is that only about 15% of the fossil-fuel emissions
remain in the atmosphere. This projects that an unabated exponential rise of global
fossil-fuel emission to the year 2050 will add about 160 GtC to the year 1860
atmospheric level of 600 GtC. The long-term atmospheric projection is quite robust
because the fraction of the fossil-fuel emission remaining in the atmosphere is
relatively insensitive to oceanic changes, as a result of the assumption that the
atmospheric residence time changes only marginally with time. As discussed above,
the conventional models assume that anthropogenic emissions face an oceanic
barrier that is of a magnitude greater than that faced by the preindustrial flux.
The following discussion deals with this key difference more fully.

3. Discussion

It is difficult to accept the conventional premise of two separable atmospheric
diffusion processes, one for the preindustrial natural base and one for the industrial
increment, occurring simultaneously for the same molecule and with the same
sinks but differing by an order of magnitude. It seems very unlikely, although not
impossible, that the capacity of the fast-acting preindustrial sinks would be so sharply
limited that they would reject the small anthropogenic addition even at its onset,
which even today is only about 4% of the estimated natural flux. Natural annual
flux variations may reasonably have been greater than this flux due to climate
instabilities. This inconsistency of the two residence times apparently has not been
clearly explained in the literature. Obviously, the carbon sinks are not easily and
completely defined, but one would expect that the dynamics of the sinks would show a smooth and continuous transition from the preindustrial era to the present.

The usual explanation of the increased resistance of the oceanic sink to absorbing anthropogenic CO\textsubscript{2} from the atmosphere is the action of the chemical buffering factor. The chemical “buffer” or “Revelle” factor determines the balance of CO\textsubscript{2} gas and Dissolved Inorganic Carbon (DIC) in the oceanic mixed layer. As noted by Siegenthaler and Oeschger\textsuperscript{9} gaseous CO\textsubscript{2} is less than 1 percent of the total DIC in surface sea-water, the main species being bicarbonate (HCO\textsubscript{3}\textsuperscript{-}) and carbonate (CO\textsubscript{3}\textsuperscript{2-}) ions. An increase in the CO\textsubscript{2} concentration shifts the chemical equilibria, as accounted for by a buffer factor $\xi$. If the CO\textsubscript{2} pressure is increased by p percent, the total CO\textsubscript{2} concentration of DIC in the seawater, increases by only p/$\xi$ percent. The factor $\xi$ increases with CO\textsubscript{2} pressure according to the Siegenthaler and Oeschger formula

$$\xi = 9 + 4.9 \frac{p_m}{p_o} - 0.1 \left(\frac{p_m}{p_o}\right)^2$$

(1)

where $p_m$ is the excess CO\textsubscript{2} pressure in the mixed layer over the preindustrial pressure $p_o$, and $p_m/p_o$ is therefore the relative increase. The size of the buffer factor determines in a crucial way the ocean’s capacity for taking up additional CO\textsubscript{2}. 

No, it Rejects Henry’s Law

The above equation provides a guide to the expected increase in the buffer factor from the preindustrial to the present era. The ratio $p_m/p_o$ is now about 0.25 [(750-600)/600], so $\xi$ should increase from a preindustrial 9 to 10.2 now, a 13% increase. A $\xi$ of 10 is now considered average, i.e. a change of 10% in atmospheric CO\textsubscript{2} produces about a 1% change in oceanic DIC (Najjar\textsuperscript{10}). A reduction in the air/sea exchange of 13% would result in a residence-time increase of $\sqrt[1.87]{15}$ or 15%. So, a 25% increase in atmospheric CO\textsubscript{2} has resulted roughly in a 15% increase in residence time. Thus, the preindustrial residence time of 4.3 years now becomes $4.3 \times 1.15 = 4.95$ years, comparable to the apparent residence time now of about 750/150=5 years. The complexity of the mixed layer ocean chemistry suggests that the above is an over-simplified calculation (Najjar\textsuperscript{10}). However, it does indicate that the probable increase in the buffering factor as a consequence of anthropogenic activity is too small to support the assumption of the oceanic saturation needed for an order of magnitude increase of residence time for atmospheric CO\textsubscript{2} from the preindustrial era.

The preceding discussion leads to the question as to whether the conventional cause/effect correlation of atmospheric CO\textsubscript{2} growth with fossil-fuel use is a misinterpreted happenstance. We know that correlation in science is not proof,
presumptive as it may appear. The fact that both the CO$_2$ and fossil-fuel emissions have grown exponentially certainly supports the conventional assumption, but it has also required adjustment of the key parameters in the various models to make them fit this assumption. Unfortunately, the complex ocean sink-absorption dynamics involves much speculation and remains quantitatively uncertain (see chapter 3 of Ref. 1). The storage pools of carbon are so large (750 GtC in the atmosphere, 38,000 in the ocean, and 2000 terrestrial) that a slow shift in their annual contribution to the natural fluxes might create a similar effect. Could such a shift in nature’s CO$_2$ source emissions be overriding man’s relatively small activities? Perhaps, for example, long-term natural transients in ocean currents or temperature distributions or deep-sea upwelling may be increasing the ocean’s emission rate. How big a change might be involved for such a hypothesis to be credible?

The basic question is whether the preindustrial 5-year residence time is applicable to current increments to atmospheric CO$_2$. There is a body of such empirical evidence. Confirming evidence is implied from the C$^{14}$ atmospheric decay data after the cessation of high-altitude nuclear weapons testing in 1964 (see Fig. 3.6 on p. 103 of Ref. 1). Because the bomb C$^{14}$ was a minimal pulse in the total atmospheric carbon, its decay characteristic in the troposphere should be indicative of the undisturbed CO$_2$ behavior. The data show an exponential decay with a C$^{14}$ residence time of about 11 years as measured at the ocean surface. However, the major part of the C$^{14}$ was emitted into the stratosphere, and there is a 5-8 year delay for its transfer into the troposphere (Ref. 1, p. 104). Thus the C$^{14}$ tropospheric residence time after 1964 must have been 3-6 years, roughly similar to that of the natural preindustrial cycle previously derived. This conclusion is a strong experimental indicator of a short residence time after 1964.

Another indicator of the existence of a short residence time is the constancy of the amplitude of the annual seasonal cycle of CO$_2$ atmospheric concentration. As shown in the Mauna Loa records, this seasonal variation was about 5.4 ppmv (± 15%) between 1960 and 1990, with a slight indication of an increase in later years. During this 30-year interval, fossil-fuel combustion emitted 148 GtC, 61% of the total of 238 GtC from 1860. During this period, the average atmospheric CO$_2$ concentration increased from 317 to 354 ppmv (50% of the change since 1860) with no indication that the seasonal variation was affected, as it would be if the residence time had changed. Some atmospheric residence time increase might have been expected from increased ocean sink saturation, but none was evident in the Mauna Loa record. An increase in atmospheric residence time would decrease the amplitude of the seasonal variation. Further, if one accepts the suggestion that forest- and soil-emitted CO$_2$ have been steadily decreasing in this century (Ref. 1, Fig. 3.18 on p. 136), then the
seasonal amplitude should also show a decrease unless the ocean emission component is correspondingly increased. In any event, the constancy of the amplitude of the seasonal variation certainly suggests no increase in bulk residence time during the 30-year period of this record. It would be useful to know the seasonal amplitude in pre-anthropogenic times to check the validity of this test for residence-time constancy.

Most significant is the magnitude of this annual swing in atmospheric CO$_2$ concentration. The Mauna Loa swing of about 5.4 ppmv is low compared to that at Point Barrow, Alaska, where it has averaged about 15 ppmv. It is generally assumed that this seasonal variation is caused by the annual photosynthesis cycle, both terrestrial and oceanic "spring bloom". The Mauna Loa annual swing is equivalent to 11 GtC/yr. The Point Barrow variation is almost three times as great. Presumably the global average would be in between. The net primary production of the terrestrial biomass has been estimated$^5$ at 57 GtC/yr out of a total annual photosynthesis of about twice this. The global marine primary production has been estimated at 27 GtC/yr.$^9$ The seasonal photosynthesis cycle needed to produce these swings can be analytically simulated by an annual sinusoidal source modulating the average flux. The mathematical analysis of the influence of such a sinusoidal source is presented in the Appendix. It is especially interesting to note that a sine source results in a cosine effect, i.e. the maximum atmospheric carbon occurs three months later than the source maximum, as has been empirically observed. As expected, the seasonal peak-to-trough swing is inversely proportional to the atmospheric residence time. For a 5-year residence time, a preindustrial 600 GtC atmospheric mass and a Mauna Loa annual swing of 11 GtC requires a source swing of 80 GtC from peak-to-trough. A 50-year residence time would require an 800 GtC source swing. Only the short residence time of about 5 years is consistent with a photosynthetic causation for the observed annual variation in atmospheric CO$_2$.

The past century's trend of changing isotopic carbon ratios does not provide insight to the fossil fuel contribution to atmospheric CO$_2$ (cf. Bolin, Ref. 1, Figs. 3.4 & 3.5, p. 101). Any carbon source from a chemical transformation involving fractionation of the isotopes C$^{12}$ and C$^{13}$ would create the same dilution of the atmospheric isotope ratios as is attributed to fossil-fuel contribution. Wood and fossil fuels are similar in this respect. The C$^{13}$ data indicate that deforestation and soil CO$_2$ may have contributed as much to the atmosphere as fossil-fuel use (Ref. 1, Fig. 3.18 on p. 136). The century's drop in C$^{14}$ of several percent does indicate sources of several thousand years age as partial contributors to the rise in atmospheric CO$_2$. This is not quantitatively inconsistent with the above fossil-fuel contribution. Based
on a review of the isotope data, Bolin (Ref. 1, p. 104-105) concludes that the mean residence time of CO₂ in the atmosphere before transfer into the sea is 8.5 ± 2 years.

Using a procedure similar to that of Segalstad and Jaworowski,¹¹ an upper bound for atmospheric residence time can be calculated from historical C¹³ data. If the entire change in δ¹³C over a period of time is attributed to fossil-fuel contributions, a residence time of roughly 12-years results. Clearly, this represents an upper bound as fossil-fuel sources are indistinguishable from natural inputs from the decay of photosynthetic materials. If half of the change in δ¹³C is attributed to these natural sources, a residence time of 5-years results.

If these arguments for a short residence time are plausible, other source terms with a growth rate similar to that of fossil-fuel combustion would be required to match the CO₂ growth. It is interesting that both global population and economic growth have similar exponential growth. Anthropogenic activity may create a variety of sources. Agriculture may be one such source. In the century 1750-1850, the conversion of forests to agriculture released about 40 GtC from trees and soils. The global population increased 5 times from 1850 to 1990. If food production followed population, an additional 200 GtC might have been released, with about the same exponential increase as fossil-fuel burning. This is a crude estimate but of the same size as the fossil-fuel emission. So, of the atmospheric increase of 150 GtC, fossil-fuel burning and food production might each account for one fifth, leaving 3/5 from another source (about 19 GtC/yr) if the 5-year residence time is valid. This is a considerable amount from unknown sources.

Is such a growth coincidence possible? Most conventional models assume that the hydrosphere flux now exchanges about 90 GtC/yr, and land sources supply about 110 GtC/yr for a total of 200 GtC/yr -- more than the 140 GtC/yr total suggested by the Sundquist preindustrial estimate³ used in the model of this study. Can these have increased annually in the incremental amounts needed to fill the above 19 GtC/yr gap? It is obvious that the relatively large size of the natural flux compared to annual fuel-use would make such an increase worth considering. How likely is it that a roughly 10% increase in natural flux would go undetected even if it occurred over 130 years? Could deep ocean volcanic action be involved? Could it be a time grouping of several increases in natural processes?

Summary

(1) The assumption that all the measured rise in atmospheric CO₂ is due to fossil-fuel use is seemingly supported by their rough timing correlation but lacks
direct experimental verification. (2) The difference in residence-time of CO₂ for the
natural flux cycle (about 5 years) and that usually attributed to anthropogenic
emission as a cause of the observed rise in atmospheric CO₂ (about 50 years ± 20%)
does not appear to have a plausible explanation. (3) The measured decay of the
atmospheric C¹⁴ from weapons testing indicates that the CO₂ residence time after
the 1964 cessation of weapons testing was about the same as the pre-anthropogenic
residence time of about 5 years. (4) The amplitude of the seasonal variation in
atmospheric CO₂ appears unchanged during 1960 to 1990, suggesting no gradual
increase in residence time during this period when 2/3 of the fossil fuel contribution
occurred and some saturation of oceanic sinks would have been expected from the
conventional models. (5) The physical size of this annual seasonal variation in CO₂
is consistent with a photosynthesis causation only if the atmospheric residence time
is short, about 5 years or so. (6) The mass balance of the carbon isotopes, C¹³ and C¹⁴,
appear to be consistent with a 5-year residence time. (7) The implication of a
constant residence time of about 5 years is that anthropogenic fuel use contributed, at
most, only 23% of the observed atmospheric CO₂ increase. (8) A quandary remains.
Either the carbon sinks have devised a process to distinguish between CO₂ sources,
or other growing atmospheric sources of carbon are more significant than fossil-fuel
use.

References


2. J. L. Sarmiento and U. Siegenthaler, “Primary Productivity and Biogeochemical


7. J. D. Aber in *Climate System Modeling*, K. E. Trenberth ed., Cambridge


The author was given permission to read the manuscript by Segalstad & Jaworowski (1993). This manuscript was in 1992 submitted to and later rejected by *Nature*, with the sole reason that "30 years of greenhouse effect research cannot be wrong".

*See instead:*


*And:*

Appendix

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Pre-Industrial State

We begin by assuming the system to be in steady state during pre-industrial times (prior to 1860). As such, the annual change in carbon content of the atmosphere can be described by the following differential equation:

\[
\frac{dM_A}{dt} = X_0 - \alpha_A M_A,
\]

where \(M_A\) represents the mass of carbon in the atmosphere at time \(t\), \(X_0\) represents the natural input of carbon to the atmosphere from oceanic and terrestrial sources, and the term \(\alpha_A M_A\) represents the mass of carbon leaving the atmosphere.

Industrial State

The addition of carbon contributions from fossil-fuel use does not significantly alter the equation. If we assume that the quantity of man-made carbon emitted each year can be approximated by the equation \(R(t) = R_0 e^{\beta t}\), where the year \(t=0\) is assumed to be 1860, we can calculate the impact of such contributions on the system.

First, however, it is useful to verify the validity of our assumption. It has been calculated by M. F. Searl (private communication) that the total amount of carbon emitted from fossil-fuels, \(I(t)\), from 1860 to 1990 by commercial and non-commercial energy use, natural gas flaring, and cement production is roughly 238 GtC. Thus, integrating our equation for annual fossil fuel emissions over this time period and equating it to \(I(t)\) yields

\[
I(t) = \int_0^t R(\tau) d\tau = \int_0^t R_0 e^{\beta \tau} d\tau = \frac{R_0}{\beta} \left( e^{\beta t} - 1 \right) = \frac{R(t)}{\beta} \left( 1 - e^{-\beta t} \right).
\]

Searl also indicates that \(R(t) = 6.27\) GtC per year in 1990. From this, we can derive from Eq. (2) that \(\beta = 0.0254\), which indicates a doubling time of 27.3 years for carbon emissions, and \(R_0 = 0.232\). A comparison between this curve fit and the actual data
points calculated by Searl can be found in Fig. 1 below. Although some error occurs in early years using this approximation, and carbon emissions will certainly not grow exponentially indefinitely, for our illustrative purposes the fit seems to be quite adequate. Adding this approximation to Eq. (1) yields

$$\frac{dM_A}{dt} = X_0 + R_0 e^{\beta t} - \alpha_A M_A.$$  \hspace{1cm} (3)

Fig. 1. Historical carbon emissions from commercial and non-commercial fossil fuels, and an exponential approximation to the data.

Integrating Eq. (3) over time yields the following equation for the mass of carbon in the atmosphere at any time $t$, for $t \geq 0$:

$$M_A(t) = \frac{X_0}{\alpha_A} + \left( N_A - \frac{X_0}{\alpha_A} \right) e^{-\alpha_A t} + \frac{R_0}{\alpha_A + \beta} \left( e^{\beta t} - e^{-\alpha_A t} \right),$$  \hspace{1cm} (4)

where $N_A$ denotes the steady-state pre-industrial carbon content of the atmosphere. That is, $M_A(0) = N_A$. Further, we note from Eq. (1) that, in steady-state, $N_A = X_0/\alpha_A$; thus, we are able to reduce Eq. (4) to the following form:

$$M_A(t) = N_A + \frac{R_0}{\alpha_A + \beta} \left( e^{\beta t} - e^{-\alpha_A t} \right).$$  \hspace{1cm} (5)
Atmospheric CO$_2$ residence time

If all of the 150 GtC increase in carbon in the atmosphere from 1860 to 1990 is assumed to have come from fossil fuel sources [i.e. $M_A(130)=750$ GtC and $N_A=600$ GtC], it is found from Eq. (5) that $\alpha_A=0.0164$. This is indicative of a 61-year atmospheric residence time.

As was pointed out in the main text, the pre-industrial atmospheric residence time is estimated to be 4.3 years based on a total atmospheric mass of 600 GtC and an annual flux of 140 GtC; and the 1990 residence time is assumed to be 5.1 years based on an atmospheric mass of 750 GtC and a flux, including fossil-fuel contributions, of 147 GtC. The effects of such a change are assumed, however, to be minor; for our purposes, a constant atmospheric residence time of 4.3 years is therefore used. Using these values, we found that the carbon content of the atmosphere increased by 35 GtC from 1860 to 1990. That is, 14.6% of the carbon remained in the atmosphere.

**Seasonal Atmospheric CO$_2$ Variations**

It has been noted in the text that the annual variations in atmospheric CO$_2$ concentration measured at Mauna Loa, Hawaii, and Point Barrow, Alaska, act to confirm the belief that a shorter (roughly 5-year) residence time is more characteristic of the flow of carbon from the atmosphere to the sinks than is a longer (roughly 50-year) residence time. From direct measurement, it is known that the annual variation in CO$_2$ concentration at Mauna Loa is about 5.4 ppmv, corresponding to an annual swing of roughly 11 GtC. At Point Barrow, the numbers are 15 ppmv and 31 GtC. The greater annual swing at Point Barrow can be accounted for by its high latitude (71.2°N) as compared to Mauna Loa (19.5°N). This higher latitude lessens the amount of mixing with the atmosphere of the southern hemisphere.

One method of analyzing this annual swing in atmospheric CO$_2$ level is to represent the seasonal change in uptake and release of carbon by land and ocean biota as a sine function of the form $A \sin \omega t$, where $A$ is the amplitude of the seasonal change produced by the biota and $\omega$ is the period of oscillation (in this case, $2\pi$ radians per year). Adding this term to Eq. (1) yields

$$\frac{dM_A}{dt} = X_0 + A \sin \omega t - \alpha_A M_A; \quad (6)$$

integrating this relation as before leads to

$$M_A(t) = N_A e^{-\alpha_A t} + \frac{X_0}{\alpha_A} \left(1-e^{-\alpha_A t}\right) + \frac{A}{\alpha_A + \omega^2} \left(\alpha_A \sin \omega t - \omega \cos \omega t + \omega e^{-\alpha_A t}\right), \quad (7)$$
which can be simplified, for large $t$ and $\omega \gg \alpha$, to

$$M_A(t) = \frac{X_0}{\alpha_A} \frac{A}{\omega} \cos \omega t. \quad (8)$$

It is important to note that, although the input wave was sinusoidal in form, the output wave is a cosine function. Thus, the accumulation of carbon in the atmosphere is roughly $90^\circ$, or $1/4$ year, out of phase with the natural uptake/release cycle of the biota.

The annual measured swing in atmospheric CO$_2$ concentration described above (let us call it $\Delta M_A$ for simplicity) is a peak to trough measurement. It is the difference between the greatest and smallest CO$_2$ concentrations measured in a single year. As such, it is equivalent to twice the amplitude of the cosine term in Eq. (8). That is, $\Delta M_A = 2A/\omega$. Noting that the average value for the atmospheric CO$_2$ concentration is $X_0/\alpha_A$ in steady state, it can be seen that

$$\frac{\Delta M_A}{M_A} = \frac{2A\alpha_A}{X_0\omega} = \frac{A\alpha_A}{X_0\pi}. \quad (9)$$

Because both $\Delta M_A$ and $M_A$ are measured values, we can use Eq. (9) to calculate the amplitude of the seasonal variation from land and sea biota, $A$, necessary to produce the measured atmospheric CO$_2$ variation, $\Delta M_A$, for a specific residence time. From the main text, we know that $X_0 = 140$ GtC/yr in steady-state. Entering this value into Eq. (9) along with $\alpha_A = 0.2$, equivalent to a 5-year residence time; $\Delta M_A = 11$ GtC from the Mauna Loa data; and $M_A = 600$ GtC yields $A = 40$ GtC. Thus, the entire peak-to-trough swing of carbon output from land and sea biota is roughly 80 GtC per year for a 5-year residence time. For a 50-year residence time and all other values the same, we find that $A = 400$ GtC and the total peak-to-trough swing is 800 GtC per year. From these calculations, then it becomes clear that only a 5-year residence time is consistent with the natural annual photosynthesis swing in atmospheric CO$_2$ levels.